LITERATURE CITED

1. A. N. Strozhkov, Sh. L. Tesler, S. P. Shabanov, et al., Treating Materials Difficult to Treat with Heat by Cutting [in Russian], Mashinostroenie, Moscow (1977).
2. V. V. Tsotskhadze, "Productive treatment of preheated heat-resistant alloys," in: Highly Productive Cutting in Machine Construction [in Russian], Mashinostroenie, Moscow (1966), pp. 71-75.
3. P. F. Bulanyi and S. P. Polyakov, "Optimization of metal heating by an electrical arc," Inzh.-Fiz. Zh., 39, No. 4, 687-691 (1980).
4. A. V. Nikolaev, "On a cascade arc in plasma installations," Avtomatich. Svarka, No. 6, 4-7 (1971).
5. P. F. Bulanyi and S. P. Polyakov, "On the question of an experimental investigation of the heat flux and electrical current densities on the anode of a high-current stabilized arc," Teplofiz. Vys. Temp., 19, No. 3, 497-501 (1981).
6. P. F. Bulanyi and S. P. Polyakov, "Investigation and generalization of the specific heat flux and current density in the anode spot of a high-current arc at atmospheric pressure," Izv. Sib. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk, No. 3, 55-57 (1980).
7. A. V. Lykov, Theory of Heat Conduction [in Russian], Vysshaya Shkola, Moscow (1967).
8. P. F. Bulanyi, S. P. Polyakov, and M. G. Rozenberg, "Investigation and generalization of the volt—ampere characteristics of direct-action plasmotrons," Inzh.-Fiz. Zh., 41, No. 2, 334-338 (1981).
9. S. V. Dresvin (ed.), Physics and Engineering of a Low-Temperature Plasma [in Russian], Atomizdat, Moscow (1972).

STOCHASTIC SELF-OSCILLATIONS IN THE PRESENCE OF DRY FRICTION

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We propose and investigate a mathematical model of a one-dimensional mechanical system which executes stochastic self-oscillations for certain values of the parameters.

Many mechanical systems involve the relative motion of bodies in the presence of dry friction. It is known [1, 2] that self-oscillations are possible in such systems, and a number of papers have been devoted to their study. There are two characteristics of the laws of dry friction, either of which can lead to the initiation of self-oscillations in an elastic system. The first is the decrease of the frictional force with increasing relative velocity [3], and the second is the increase of the static frictional force with an increase in the time of stationary contact [4, 5].

In the present article we propose a simple model of a mechanical system in which stochastic self-oscillations occur for certain values of the parameters.

We consider a body moving on a plane under the action of a spring of stiffness c whose end is displaced with a constant velocity v. We assume that the force of sliding friction has the constant value F_0, and that the maximum force of static friction $F(\tau)$ depends on the time τ of stationary contact between the body and the plane in the following way: $F(0) = F_0$, and $F(\tau)$ increases monotonically with increasing τ and approaches a finite value F_1 as $\tau \to \infty$.

The behavior of such a system was investigated in [4, 5] for

$$F(\tau) = F_1 - (F_1 - F_0) e^{-\delta\tau}, \tag{1}$$

but the parameters F_1, F_0, and δ were assumed variable over rather narrow limits. We show below that the whole (F_1, δ) plane is divided into a number of nonintersecting domains, one

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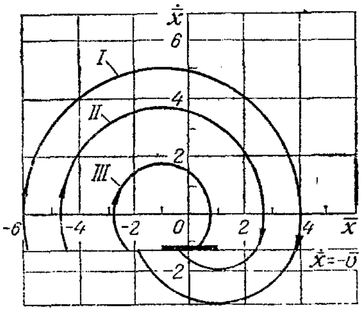

Fig. 1

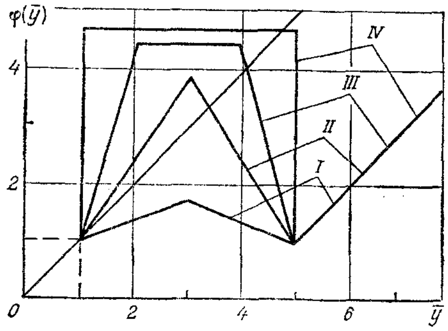

Fig. 2

Fig. 1. Motion of representative point as a function of the coordinate of the point of separation: I) $\bar{x}_n < -5$; II) $-5 < \bar{x}_n < -3$; III) $-3 < \bar{x}_n < -1$.

Fig. 2. Possible types of mapping function. I) for $k < 0.5$; II) for $f_1 > 3$, $0.5 < k < (f_1 - 1)/(f_1 + 1)$ corresponds to stochastic self-oscillations; III) for $0.5 < k < 1$; $k > (f_1 - 1)/(f_1 + 1)$; IV) for $k > 1$.

of which corresponds to the values of the parameters for which stochastic self-oscillations are initiated.

The equation of motion of the body has the form

$$m\ddot{x} = F_{FR}(\dot{x}) - c(x - vt), \tag{2}$$

where for $\dot{x} > 0$ $F_{FR}(\dot{x}) = -F_0$, for $\dot{x} < 0$ $F_{FR}(\dot{x}) = F_0$, and for $\dot{x} = 0$ the frictional force is equal to the spring tension, as long as the tension does not exceed the maximum force of static friction $F(\tau)$.

Transforming to dimensionless variables by the formulas

$$x - vt = \frac{F_0}{c}\bar{x};\quad t = \sqrt{\frac{m}{c}}\,\bar{t};\quad \frac{F}{F_0} = f, \tag{3}$$

we reduce Eq. (2) to the form

$$\ddot{\bar{x}} + \bar{x} = f_{FR}(\dot{\bar{x}} + \bar{v}). \tag{4}$$

We investigate the phase trajectories of Eq. (4) in the $(x, \dot{\bar{x}})$ plane (Fig. 1).

For $\dot{\bar{x}} > -\bar{v}$ the force $f_{FR}(\dot{\bar{x}} + \bar{v}) = -1$, and consequently the phase trajectory will be an arc of a circle with its center at the point $(-1, 0)$. For $\dot{\bar{x}} < -\bar{v}$, when $f_{FR}(\dot{\bar{x}} + \bar{v}) = 1$, the phase trajectory will be an arc of a circle with its center at the point $(1, 0)$. If the representative point falls on the segment L (it is determined by the conditions $\dot{\bar{x}} = -\bar{v}$ and $|\bar{x}| < 1$), it "sticks" to it, since on this segment the force of sliding friction exceeds the spring force which causes the motion, and the body is at rest with respect to the plane, but moves with a velocity $\dot{\bar{x}} = -\bar{v}$ relative to the end of the spring. The coordinate $\bar{x}_1$ at which the representative point separates from the straight line $\dot{\bar{x}} = -\bar{v}$ and begins to move in a circle depends on the maximum force of static friction, which in turn is determined by the time of stationary contact, i.e., the time the representative point moves along the straight line $\dot{\bar{x}} = -\bar{v}$. Clearly we have for $\bar{\tau}$

$$\bar{\tau} = \frac{\bar{x}_0 - \bar{x}_1}{\bar{v}}. \tag{5}$$

Thus, for $\bar{x}_1$ we obtain the equation

$$\bar{x}_1 = -f\left(\frac{\bar{x}_0 - \bar{x}_1}{\bar{v}}\right), \tag{6}$$

from which $\bar{x}_1$ is determined as a function of $\bar{x}_0$:

$$\bar{x}_1 = \varphi(\bar{x}_0). \tag{7}$$

We proceed to the construction of the map of the track for the coordinates $\bar{x}_n$ of the separation of the representative point from the straight line $\dot{\bar{x}} = -\bar{v}$. If $\dot{\bar{x}}_n < -5$, the phase trajectory intersects the straight line $\dot{\bar{x}} = -\bar{v}$ at the point $\bar{x}_{n+1} = \bar{x}_n + 4$, and leaves the segment L. For $-5 < \bar{x}_n < -3$ the representative point falls on the segment L at the coordinate $\bar{x}_n' = \bar{x}_n + 4$, in terms of which $\bar{x}_{n+1}$ is expressed in accord with (7). If $-3 \le \bar{x}_n < -1$, the phase trajectory intersects the segment L at the point $\bar{x}_n' = -\bar{x}_n - 2$, and then $\bar{x}_n'$ goes over to $\bar{x}_{n+1}$ in accord with (7).

Thus, we obtain the following map of the track

$$\bar{x}_{n+1} = \begin{cases} \bar{x}_n + 4 \text{ for } \bar{x}_n < -5; \\ \varphi(\bar{x}_n + 4) \text{ for } -5 < \bar{x}_n < -3; \\ \varphi(-\bar{x}_n - 2) \text{ for } -3 < \bar{x}_n < -1. \end{cases} \tag{8}$$

The map of the track is an exceptionally convenient method for studying the phase-plane diagram of a system; it gives an adequate description of the nature of the motion and its classification. The mapping of the track by piecewise-linear and quadratic functions is described in detail in [6]. Even in the relatively simple case of a quadratic function, a dynamic system with such a mapping function admits, in addition to a simple oscillatory process and random self-oscillations, a whole series of intermediate motions which are a sequence of cycles. Each cycle consists in turn of a sequence of oscillations with various amplitudes and periods, the number of which may be arbitrarily large, depending on the specific values of the parameters. Therefore, it is expedient to replace the original relation for $F(\tau)$ described by Eq. (1) by a piecewise-linear function in qualitative agreement with (1). With such a simplification a number of intermediate admissible motions drop out of consideration, but the basic bifurcations on the path from harmonic to random oscillations remain. Thus, we take $F(\tau)$ as follows:

$$F(\tau) = \begin{cases} F_0 + a\tau \text{ for } \tau \leqslant \dfrac{F_1 - F_0}{a}; \\ F_1 \text{ for } \tau \geqslant \dfrac{F_1 - F_0}{a}. \end{cases} \tag{9}$$

Then the function $\varphi(\bar{x}_n)$ takes the form

$$\varphi(\bar{x}_n) = \begin{cases} -\dfrac{1 + k\bar{x}_n}{1 - k} \text{ for } \dfrac{k}{1-k} \leqslant \dfrac{f_1 - 1}{1 + \bar{x}_n}, \ k < 1, \\ f_1 \text{ for } \dfrac{k}{1-k} \geqslant \dfrac{f_1 - 1}{1 + \bar{x}_n}, \ k < 1, \end{cases} \tag{10}$$

where $k = a/vc$ is a dimensionless parameter equal to the rate at which the function $f((\bar{x}_0 - x_1)/\bar{v})$ increases with increasing $\bar{x}_0$ from -1 to 1. If $k > 1$, the function $\varphi(\bar{x})$ reaches a maximum value f_1 by a jump at any $\bar{x}$ from the interval $|\bar{x}| < 1$.

Figure 2 shows graphs of the mapping function for certain f_1 and k, where for convenience we have made the change of variables $\bar{y} = -\bar{x}$, since the coordinates $\bar{x}_n$ of the point of separation take on only negative values: $\bar{x}_n < -1$. Under the condition $f_1 > 3$, $0.5 < k < 1$, and $k < (f_1 - 1)/(f_1 + 1)$, the mapping function has the form II (Fig. 2), which corresponds to stochastic self-oscillations [6, 7].

Figure 3 shows the division of the (f_1, k) plane into nonintersecting domains which describe the bifurcations of the system under study.

If the parameters f_1 and k belong to domain 1, the point $\bar{y} = 1$ is stable (Fig. 2), and the system executes harmonic oscillations of a definite character: when the amplitude increases by chance, it again decreases to the initial value in the course of time; if the amplitude decreases by chance, it remains without change until the next chance action.

If the parameters f_1 and k belong to domain 2, the stable form of motion will be relaxation oscillations with constant amplitude and frequency.

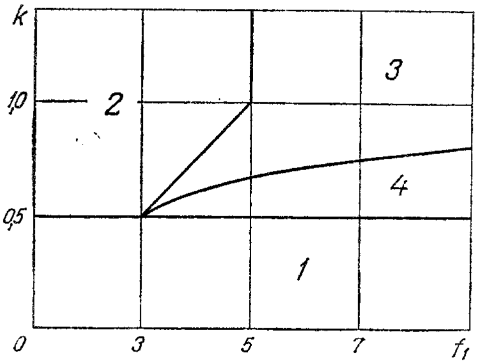

Fig. 3

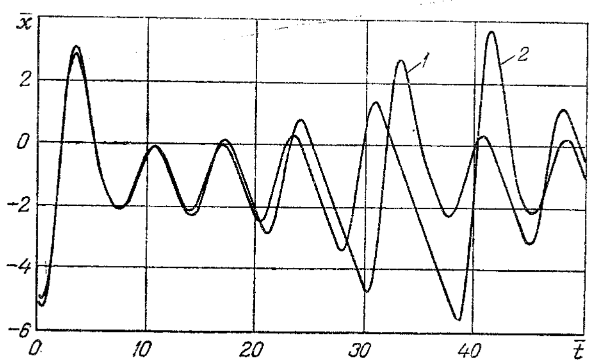

Fig. 4

Fig. 3. Division of the plane of the parameters into domains: 1) harmonic self-oscillations; 2) relaxation self-oscillations; 3) parameters for which the system executes a sequence of relaxation self-oscillations; 4) stochastic self-oscillations.

Fig. 4. Time dependence of coordinate of body during stochastic self-oscillations ($f_1 = 7$, $k = 0.7$). Curves 1 and 2 are for different initial coordinates, $\bar{x}$, but the same initial derivatives $\dot{\bar{x}}$.

When the parameters belong to domain 3, the motion of the system breaks up into a sequence of identical cycles, each of which in turn consists of a finite sequence of oscillations with various amplitudes and frequencies.

Finally, domain 4 corresponds to random self-oscillations. In other words, the amplitude and frequency of the self-oscillations vary in a random manner within limits fixed by the values of the parameters f_1 and k.

Figure 4 shows two curves for $\bar{x}(\bar{t})$ for the same initial velocities, but slightly different initial coordinates. It is clear from the figure that as $\bar{t}$ increases, the curves diverge in spite of nearly the same initial values.

NOTATION

c, spring stiffness; F_0, force of sliding friction; F_1, maximum force of static friction; $f_1 = F_1/F_0$, dimensionless force; $F_{FR}(\dot{x})$, force of friction as a function of velocity; m, mass of body; t, time; $\bar{t}$, dimensionless time; x, coordinate of body; $\bar{x}$, dimensionless coordinate of body; v, velocity of end of spring; $\bar{v} = vF_0/\sqrt{mc}$, dimensionless velocity of end of spring; τ, time of stationary contact between body and plane; $\bar{\tau}$, dimensionless time of stationary contact between body and plane.

LITERATURE CITED

1. A. A. Andronov, A. A. Vitt, and S. É. Khaikin, The Theory of Vibrations [in Russian], Nauka, Moscow (1981), pp. 257-268.
2. Vibrations in Engineering (Handbook) [in Russian], Vol. 2, Mashinostroenie, Moscow (1979), pp. 172-173.
3. N. L. Kaidanovskii and S. É. Khaikin, "Mechanical relaxation oscillations," Zh. Tekh. Fiz., 3, 91 (1933).
4. A. Yu. Ishlinskii and I. V. Kragel'skii, "Jumps in the presence of friction," Zh. Tekh. Fiz., 14, 276 (1944).
5. Yu. I. Kosterin, Mechanical Self-Oscillations in the Presence of Dry Friction [in Russian], Izd. Akad. Nauk SSSR (1960).
6. E. Ott, "Strange attractors and chaotical motion of dynamical systems," Rev. Mod. Phys., 53, 655 (1981).
7. M. I. Rabinovich, "Stochastic self-oscillations and turbulence," Usp. Fiz. Nauk, 125, 123 (1978).